注意：一、務請依序作答，否則酌予扣分.
　　　二、不抄題，但須標明題號.
　　　三、每題均須寫出計算過程或說明道理，否則不予計分.

1. (1) Let $f : [0, +\infty) \longrightarrow \mathbb{R}$ such that $f(x) = \cos \sqrt{x}$.
   (a) Find the derivative of f;　　　　　　　　　　4%
   (b) Evaluate $\int \cos \sqrt{x}\, dx$.　　　　　　　　4%

   (2) Find the limit $\lim_{x \to 0+} x^{\tan x}$.　　　　　　8%

2. (1) Find the Maclaurin series of $f(x) = (1+x)^\alpha$, where $\alpha \in \mathbb{R}$. Show that
   the function $f(x)$ is analytic at $x = 0$.　　　8%

   (2) Find the 4-th order Taylor's expansion of $\sin(x + 2y)$ at the point $(0,0)$.　　8%

3. (1) Find $\lim_{(x,y) \to (0,0)} \dfrac{\sin(x^2 + y^2)}{x^2 + y^2}$.　　　　8%

   (2) Is the function

   $$f(x,y) = \begin{cases} (x^2 + y^2)\sin \dfrac{1}{\sqrt{x^2 + y^2}}, & \text{if } (x,y) \neq (0,0), \\[2mm] 0, & \text{if } (x,y) = (0,0), \end{cases}$$

   continuously partially differentiable at $(0,0)$. Is f differentiable at $(0,0)$?　　8%

4. (1) Show that $\displaystyle\int_0^{+\infty} e^{-x^2}\, dx$ is convergent. Also find the value which the improper
   integral converges to.　　　　8%

   (2) Let R be the region between the graph of the curve $y = \exp(-x^2)$ and its
   asymptote. Find the volume of the solid generated by revolving the region R
   about the Y-axis.　　　　8%

5. (1) Let $a_1 > 0$, $a_{n+1} = \dfrac{6(1 + a_n)}{7 + a_n}$. Show that the sequence $\{a_n\}$ is convergent
   and find its limit.　　　　8%

   (2) Is the series $\displaystyle\sum_{n=1}^{+\infty} \dfrac{e^n n!}{n^n}$ convergent? If it is convergent, find its sum; otherwise,
   show the reason why it is divergent.　　　　10%

6. (1) Let R be a connected compact region in $\mathbb{R}^2$ and let γ be the boundary of R
   such that it is a smooth closed oriented simple curve. Show that the area of
   R is

   $$\frac{1}{2}\int_\gamma -y\, dx + x\, dy.$$

   　　　　8%

   (2) Consider the extrema of the function $f(x,y) = (y - x^2)(y - x^3)$.　　　10%

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