PhD Qualify Exam. Analysis Mar. 4, 2016

(E: easy, M: moderate, D: difficult)

1. (E, 15 pts, 2010, 3) Show that measurable sets are those which split every set (measurable or not) into pieces that are additive with respect to outer measure.

2. (E, 15 pts, 2007, 9) Show that every Lebesgue integrable function is the limit, almost everywhere, of a certain sequence of step functions.

3. (M, 20 pts, 2007, 9) $1 < p < \infty$, $f \in L^p(0,\infty)$. Define

$$F(x) = \frac{1}{x}\int_0^x f(t)dt, \quad 0 < x < \infty$$

   (a) Prove that

$$\|F\|_p \leq \frac{p}{p-1}\|f\|_p$$

   (b) Prove that the equality holds only if $f = 0$ a.e..

4. (M, 20 pts, 2010, 9) Let f be a bounded function on the closed, bounded interval $[a,b]$. Then f is Riemann integrable over $[a,b]$ if and only if the set of points in $[a,b]$ at which f fails to be continuous has measure zero.

5. (E, 15 pts) Prove the Minkowski inequality for $0 < p < 1$.

6. (E, 15 pts) Show that a normed linear space is complete if and only if every absolutely summable series is summable.