

離散數學

EACH OF THE FOLLOWING TEN PROBLEMS IS TEN POINTS AND THE TOTAL IS 100 POINTS.

1. Is it true that any problem that can be solved with a recursive algorithm can be solved with a non-recursive algorithm using repetition, and vice versa? Give an example to indicate that your answer is corrected.
2. What are the generally accepted characteristics of a high-quality software? Explain each of them briefly.
3. Give an algorithm to solve the eight-queen problem. (That is to find all ways to place eight nontaking queens on an 8×8 chessboard. One queen can capture another queen if they are both in the same row or the same column or on a common diagonal.)
4. Is $K_{3,3}$ an interval graph, or a unit interval graph? Please give the reasons for each of your answers.
5. Is it true that two graphs are isomorphic if and only if their complementary graphs are isomorphic? Why? Also give an example to show your answer is corrected.
6. Show that every planar graph can be 5-colored.
7. Give examples to explain the advantages for using the depth-first search and the breadth-first search, respectively.
8. Let $h(x)$ be a generating function with a_r the coefficient of x^r . Show that $g(x) = h(x)/(1-x)$ is a generating function of the sums of the a_r 's. Use this result to evaluate the sum
$$3 \cdot 2 \cdot 1 + 4 \cdot 3 \cdot 2 + 5 \cdot 4 \cdot 3 + \dots + (n+1)n(n-1).$$
9. Suppose that an elf has a staircase of n stairs to climb. Each step it can cover either one stair or two stairs. Find a recurrence relation for $a(n)$, the number of different ways for the elf to ascend the n -stair staircase. Also solve $a(n)$ in terms of n .
10. Explain the advantages in using the balanced binary tree search?