MIDTERM FOR GEOMETRY

Date: Wednesday, April 25, 2001
Instructor: Shu-Yen Pan

No credit will be given for an answer without reasoning.

1. Consider the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.

(i) [5%] Find the unit tangent vector $\mathbf{t}$ at $(0,0,0)$.

(ii) [5%] Find an equation of the osculating plane at $(0,0,0)$.

2.

(i) [5%] Is it possible that a differentiable curve whose curvature is zero in some interval but its torsion is not zero in that interval? Why or Why not? On the other hand, is it possible that a differentiable curve whose torsion is zero in some interval but its curvature is not zero in that interval? Why or Why not?

(ii) [5%] Give examples of two curves with the same curvature but different torsion in some interval.

3. Knowing that $g_{11} = 1$, $g_{12} = g_{21} = 0$ and $g_{22} = \cos^2(u^1)$. Compute:

(i) [5%] $g_{ij}g^{jk}$

(ii) [5%] $\left(\frac{\partial}{\partial u^j} g_{kl}\right) g^{jk}$

4. [10%] Let f and h be two differentiable functions of one variable. Compute the first fundamental form of the surface of revolution:

$$x = f(u)\cos v, \qquad y = f(u)\sin v, \qquad z = h(u).$$

5. [10%] Compute the area of the helicoid

$$x = u\cos v, \qquad y = u\sin v, \qquad z = 2v$$

for $0 \le u \le 1$ and $0 \le v \le 2\pi$.

6. Let the helicoid be as in problem 5. Compute:

(i) [5%] b_2^1 at $(1,0,0)$.

(ii) [5%] Γ_{122} at $(1,0,0)$.

7. [10%] Let the helicoid be as in problem 5. Find an equation of the tangent plane at $(1,0,4\pi)$

8. [10%] Let $\mathbf{r}(u^1,u^2)$ be a regular surface. Let $\mathbf{m}$ be the unit normal vector of the surface. Show that $\mathbf{m}_i$ can be written as a linear combination of $\mathbf{r}_1$ and $\mathbf{r}_2$.

9.

(i) [5%] What is the Gaussian curvature K at the point $(0,0,1)$ on the surface $x^2+y^2+z^2=1$?

(ii) [5%] What is the Gaussian curvature K at the point $(\sqrt{3},\sqrt{3},\sqrt{3})$ on the surface $x^2+y^2+z^2=9$?

10. [10%] Give an example of a differentiable curve whose curvature (as a function of a parameter) can take any positive real values.