學號: Name:

Part I. (30%) Multiple-Choice Questions (Select one or more answer choices each question)

1. Which of the following limit is 1?
(A) $\lim_{x\to 0^+} \frac{|x|}{x}$ (B) $\lim_{x\to 0} \frac{\sec x - 1}{x}$ (C) $\lim_{x\to 0} \frac{\tan^2 x}{x^2}$ (D) $\lim_{x\to 0} e^{-x^2}$
(E) $\lim_{x\to 0^+} \ln x$
Answer:

2. Which of the following is asymptote of the graph $y = \frac{x+2}{\sqrt{x^2-9}}$?
(A) $x = 3$ (B) $x = -3$ (C) $x = -2$ (D) $y = 1$ (E) $y = 0$
Answer:

3. Which of the following function is differentiable everywhere?
(A) $f(x) = |x|$ (B) $f(x) = \sqrt[3]{1 + e^{\cos x}}$ (C) $f(x) = \int_0^{\cos x} t \arcsin t \, dt$
(D) $f(x) = \begin{cases} x^3, & \text{if } x \leq 1 \\ 3x - 2, & \text{if } x > 1 \end{cases}$ (E) $f(x) = \begin{cases} x \sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$
Answer:

4. Let $a = \arccos(-0.7)$. Which of the following is positive number?
(A) a (B) $\tan a$ (C) $\sec a$ (D) $\cos(2a)$ (E) $\ln|\csc a|$
Answer:

5. Which of the following series is converged?

(A) $\sum_{n=2}^{\infty} n^{-\pi}$ (B) $\sum_{n=2}^{\infty} n\pi^{-n}$ (C) $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ (D) $\sum_{n=2}^{\infty}(-1)^n n\sin(\frac{1}{n})$ (E) $\sum_{n=2}^{\infty} \frac{e^n}{n!}$

Answer:

6. Let $0 \leq a < b, f' : [a,b] \to \mathbb{R}$ be continuous function and $f(x) > 0$ on $[a,b]$, let Ω be theregion between $y = f(x)$, $x = a$, $x = b$ and $y = 0$. Which of the following statement is always right?

(A) The length of the graph f is given by $\int_a^b \sqrt{x^2 + [f(x)]^2}\, dx$.

(B) The area of the surface generated by revolving the curve f about the x-axis is given by $\int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx$.

(C) The area of the surface generated by revolving the curve f about the y-axis is given by $\int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}\, dx$.

(D) The volumn of the solid generated by revolving Ω about the x-axis is given by $\int_a^b 2\pi f(x)\, dx$.

(E) The volumn of the solid generated by revolving Ω about the x-axis is given by $\int_a^b \pi[f(x)]^2\, dx$.

Answer:

Part II.

1. (10 %) Find the minimum distance between the point $(1, 1)$ to the parabola $y = 2x^2$。

2. (8 %) Suppose $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$, prove that $\tan b - \tan a \geq b - a$.

3. Let

$$f(x) = \int_0^{\frac{1}{x}} \frac{t^2}{t^4+1} dt + \int_0^x \frac{1}{t^4+1} dt, \quad x \neq 0$$

a. (4 %) Find $f'(x)$.

b. (4 %) Find $f(1) + f(-1)$. (Hint: $\int \frac{1}{t^4+1} dt$ cannot be computed, you should use another method.)

c. (6 %) Using above, find $f(3) + f(-2)$.

4. (10 %) Let

$$f(x) = \frac{\ln x}{x}, \quad x > 0.$$

a. Find the maximum value of $f(x)$.

b. Using above, prove that $\pi^e < e^\pi$.

5. (10 %) Find the integral:

$$\int \sin(\sqrt[3]{x})\, dx$$

6. (8 %) Find the limit $\lim_{x\to 0^+} x(\ln x)^{\frac{5}{3}}$.

7. (10 %) Let $f(x) = \cos(x^2)$. Find $f^{(2014)}(0)$. (Hint: Use the Maclaurin series of $\cos x$)