

## MIDTERM 1 FOR CALCULUS

**Time:** 09:10–11:00, Tuesday, Nov 19, 2002

**Instructor:** Shu-Yen Pan

*No credit will be given for an answer without reasoning.*

1. (1) [5%] Find an equation of the tangent line to the curve  $y = \frac{x}{2-x}$  at the point  $(1, 1)$ .
  - (2) [5%] Find the limit  $\lim_{s \rightarrow 16} \frac{4-\sqrt{s}}{s-16}$ .
2. (1) [5%] Suppose that  $f$  and  $g$  are functions that are differentiable at  $x = 1$  and that  $f(1) = 2$ ,  $f'(1) = -1$ ,  $g(1) = -2$  and  $g'(1) = 3$ . Find the value  $h'(1)$  where  $h(x) = \frac{xf(x)}{x+g(x)}$ .
  - (2) [5%] Find  $\frac{dp}{dt}$  if  $p = \sqrt{x + \sqrt{x}}$ .
3. (1) [5%] Given the graph of  $y = f(x)$  below, sketch the graph of  $y = f'(x)$ .

- (2) [5%] A city's population (in thousands)  $t$  years from now is estimated to be

$$P(t) = \frac{25t^2 + 125t + 100}{t^2 + 5t + 40}.$$

At what rate will the population be increasing 10 years from now?

4. (1) [5%] Use second derivative test to find the relative extrema of the function  $g(x) = x^2 + \frac{1}{x}$ .
  - (2) [5%] Use the definition of derivative to prove that the function  $f(x) = x^{3/5}$  is not differentiable at  $x = 0$
5. (1) [5%] Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } x > 1 \end{cases}$$

Find the values of  $a$  and  $b$  so that  $f$  is continuous and has a derivative at  $x = 1$ .

- (2) [5%] When living things die, the carbon 14 in their body decays exponentially into ordinary carbon. The proportion of carbon 14 remaining after  $t$  years is  $e^{-kt}$ . We know that the half-life of carbon 14 is 5770 years. Find the constant  $k$ .
6. [10%] Find the first and second derivatives (i.e.,  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ ) of the function defined implicitly by the equation  $\sqrt{xy} = x + y$ .
7. [10%] Find the dimensions of the top-open cylindrical tin can with volume  $60\pi$  cubic centimeters that can be made from the least amount of tin.
8. [10%] Which is better? 10.2% interest compounded annually, 10% interest compounded quarterly, or 9.8% interest compounded continuously?
9. [20%] Sketch the graph of  $y = \frac{x}{\sqrt{x^2+1}}$ . (You have to provide the following information: domain,  $x$  and  $y$  intercepts, vertical and horizontal asymptotes, the intervals where the function is increasing, decreasing, concave up, concave down, local extreme values, inflection points.)