MIDTERM 2 FOR ALGEBRA

Date: 1999, November 29, 10:10–11:00AM

Each of Problems 1–5 is worth 14 points, each of problems 6-7 is worth 15 points.

1. Give an example of a group G such that $|G| = 12$ and G is not abelian.

2. Let G, H be two groups. Suppose that M is a normal subgroup of G and N is a normal subgroup of H. Show that $M \times N$ is a normal subgroup of $G \times H$.

3. Find kernel of ϕ and $\phi(14)$ for $\phi\colon \mathbf{Z}_{24} \to S_8$ where $\phi(1) = (25)(1467)$.

4. Show that the commutator subgroup of S_n is contained in A_n. (Hint: consider the homomorphism $\phi\colon S_n \to \mathbf{Z}_2$ by $\phi(\sigma) = 1$ if σ is odd and $\phi(\sigma) = 0$ if σ is even.)

5. Find a composition series of $S_3 \times \mathbf{Z}_2$.

6. Let G be the group $\langle \mathbf{R}, + \rangle$ and $X = \mathbf{R}^2$. Let $\phi\colon G \times X \to X$ be defined by

$$\phi(t, (r\cos\theta, r\sin\theta)) = (r\cos(\theta + t), r\sin(\theta + t)).$$

Show that X is a G-set via the map ϕ. Let $P = (1, 0) \in X$. Find the isotropic subgroup G_P.

7. Let K and L be normal subgroups of G with $K \vee L = G$ and $K \cap L = \{e\}$. Show that $G/K \simeq L$.