MIDTERM 1 FOR ALGEBRA

Date: 1999, October 25, 10:10–11:00AM

Each Problem is worth 15 points.

1. Let F be the set of all real-valued functions having the set $\mathbf{R}$ of all real numbers as domain. Define the operation "$\circ$" on F by $(f \circ g)(x) = f(g(x))$ for $f, g \in F$. Show that the operation "$\circ$" is associative.

2. Let $\langle S, * \rangle$, $\langle S', *' \rangle$ $\langle S'', *'' \rangle$ be three sets with binary operations. Suppose that $\phi\colon S \to S'$ and $\psi\colon S' \to S''$ are both isomorphisms. Show that $\psi \circ \phi$ is an isomorphism of $\langle S, * \rangle$ and $\langle S'', *'' \rangle$.

3. Let G be a group and let a be a fixed element of G. Show that the subset

$$H_a = \{ x \in G \mid xa = ax \}$$

is a subgroup of G.

4. Let H be the subgroup of $\mathbf{Z}_{30}$ generated by 25. Find $|H|$.

5. Let $\sigma = (1245)(36)$ in S_6. Find the index of $\langle \sigma \rangle$ in S_6.

6. Let G be a group of order pq, where p and q are prime number. Show that every proper subgroup of G is cyclic.

7. Let $\langle G, \cdot \rangle$ be a group. Consider the new binary operation $*$ on G defined by

$$a * b = b \cdot a$$

for $a, b \in G$. Then $\langle G, * \rangle$ is a group (you don't need to check this). Show that $\langle G, * \rangle$ is isomorphic to $\langle G, \cdot \rangle$.